Math 1552: Integral Calculus
Review Problems for the Final Exam
Summer 2021

******PLEASE NOTE: ******
In preparing for the final exam, you should review the following problems from our past study guides, and the additional problems on volumes listed below. You will also find some review problems from previous sections incorporated in the problems below. While the list of sections above does not include every integration technique, please note that students are expected to also understand all of the techniques we have seen in the class, and how to combine them in a single problem. This means, for example, that you may have to apply a substitution followed by another method in an integration problem, or apply L'Hopital's rule to evaluate an improper integral, as we have seen examples of throughout the course.

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1 Review of Sections 8.4–8.5, 4.5, 8.8, 10.1–10.2

Content Recap

(a) To apply L'Hopital's rule, the limit must have the indeterminate form ____________ or ____________.

(b) An integral $\int_a^b f(x)dx$ is *improper* if at least one of the limits of integration is ____________, or if there is a ____________ ____________ on the interval $[a, b]$.

(c) If we would evaluate an integral using *trig substitution*, the integral should contain an expression of one of these forms: ____________, ____________, or ____________.

Write out the trig substitution you would use for each form listed above.

(d) To use the method of *partial fractions*, we must first factor the denominator completely into ____________ or ____________ ____________ terms.

In the partial fraction decomposition, if the term in the denominator is raised to the kth power, then we have ____________ partial fractions.

For each linear term, the numerator of the partial fraction will be ____________.

For each irreducible quadratic term, the numerator will be _____________.

(e) Define the least upper bound and greatest lower bound of a sequence.

(f) What does it mean for a sequence to be monotonic?

(g) A sequence **converges** if _____________ and **diverges** if _____________.

(h) If a sequence is _____________ and _____________, then it converges.

(i) A geometric series has the general form _____________.
The series converges when _____________ and diverges when _____________.

(j) The harmonic series has the general form _____________, and it always _____________!

(k) To find the sum of a telescoping series, we should first break it into _____________ _____________.

(l) The series $\sum a_n$ diverges if the limit is NOT equal to _____________.

Problems Similar to the Studio Worksheets

2. Determine if the following statements below are always true or sometimes false.

(a) If an integral contains the term $a^2 + x^2$, we should use the substitution $x = a\sec\theta$.

(b) The expression $\tan\left(\sin^{-1}(x)\right)$ cannot be simplified.

(c) When using a trig substitution with a term of the form $a^2 - x^2$, we could use either $x = a\sin\theta$ or $x = a\cos\theta$ and obtain equivalent answers (that may differ only by a constant).

(d) If we use the trig substitution $x = \sin\theta$, then it is possible that $\sqrt{1-x^2} = -\cos\theta$.

(e) The partial fraction decomposition of $\frac{x}{(x+3)^2}$ is $\frac{A}{x+3} + \frac{B}{(x+3)^2}$.

(f) $\int \frac{dx}{(x+3)^2} = \ln(x+3)^2 + C$.

(g) The integral $\int \frac{x}{x^2-9}dx$ could be best evaluated using the method of partial fractions.

(h) The integral $\int \frac{dx}{x(x^4+1)}$ cannot be evaluated using the method of partial fractions.

(i) $\lim_{x\to\infty} xe^x$ has the indeterminate form ∞^∞.

(j) $\lim_{x\to 0^+}(\cos x)^{\frac{1}{x}}$ has the indeterminate form 1^∞.

(k) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x} = 2e$.

(l) When evaluating a limit using L'Hopital's rule, we first need to find $\left(\frac{f}{g}\right)'$.

(m) If f has a vertical asymptote at $x=a$, then $\int_a^b f(x)dx = \lim_{c\to a^+}\int_c^b f(x)dx$.

(n) $\int_{-1}^{1} \frac{1}{x} dx = 0$.

(o) Saying that an improper integral converges means that the integral must evaluate to a finite number.

(p) Indefinite integrals can be improper.

(q) If $\{a_n\}$ is bounded, then it converges.

(r) If $\{a_n\}$ converges, then it is monotonic.

(s) An unbounded sequence diverges.

(t) If $\{a_n\}$ diverges, then $\lim_{n\to\infty} a_n = \infty$.

(u) The sequence $\{\frac{1}{n}\}$ converges to 0.

(v) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ converges.

(w) If $\lim_{n\to\infty} a_n = 0$, then the sequence $\{a_n\}$ converges.

(x) The series $\sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges.

(y) The sum of two divergent series also diverges.

(z) The series $\sum_{n=1}^{\infty} r^n$ converges to $\frac{1}{1-r}$ if $|r|<1$.

3. Evaluate the following integrals using any method we have learned so far:
u-substitutions, integration by parts, integrating trig functions, trigonometric substitutions, or partial fractions.

(a) $\int \frac{x^2}{(x^2+4)^{3/2}} dx$

(b) $\int (x^2+1)e^{2x} dx$

(c) $\int \frac{\sqrt{1-x^2}}{x^4} dx$

(d) $\int \frac{dx}{e^x\sqrt{e^{2x}-9}}$

(e) $\int \sin^2(x)\cos^2(x)dx$

(f) $\int \frac{x+3}{(x-1)(x^2-4x+4)}dx$

(g) $\int x^5 \ln(x)dx$

(h) $\int \frac{x+4}{x^3+x}dx$

(i) $\int \sqrt{25-x^2}dx$

(j) $\int \frac{x-1}{(x+1)^3}dx$

(k) $\int \frac{x+2}{x+1}dx$

(l) $\int \frac{x+1}{x^2(x-1)}dx$

4. Evaluate the following limits using L'Hopital's Rule.

(a) $\lim_{x\to 0^+}[x(\ln(x))^2]$

(b) $\lim_{x\to\infty}(x+e^x)^{2/x}$

(c) $\lim_{x\to\frac{\pi}{2}}\left[\frac{\ln(\sin x)}{(\pi-2x)^2}\right]$

5. Evaluate the improper integrals if they converge, or show that the integral diverges.

(a) $\int_0^3 \frac{x}{(x^2-1)^{2/3}}dx$

(b) $\int_0^\infty x^2 e^{-2x}dx$

(c) $\int_1^4 \frac{dx}{x^2-5x+6}$

6. For each sequence below, find the l.u.b. and g.l.b., and determine if the sequence is monotonic.

(a) $\{\sin(n\pi)\}$

(b) $\left\{(-1)^{n+1}\frac{1}{5^n}\right\}$

(c) $\left\{\frac{n+1}{n}\right\}$

7. Determine whether or not each sequence converges. If so, find the limit.

(a) $\left\{\frac{2n^2}{\sqrt{9n^4+1}}\right\}$

(b) $\left\{\left(1-\frac{1}{8n}\right)^n\right\}$

(c) $\left\{\frac{n!}{e^n}\right\}$

(d) $\left\{\left(\frac{n}{n+5}\right)^n\right\}$

8. Use series to write the repeating decimal 0.31313131... as a rational number.

9. Find the sum of each convergent series below, or explain why the series diverges.

(a) $\sum_{k=7}^{\infty} \frac{1}{(k-3)(k+1)}$

(b) $\sum_{k=0}^{\infty} (-1)^k$

(c) $\sum_{k=2}^{\infty} \frac{2^k+1}{3^{k+1}}$

(d) $\sum_{k=1}^{\infty} \frac{5k^2+8}{7k^2+6k+1}$

(e) $\sum_{n=0}^{\infty} \left(\frac{2}{3^n} + \frac{(-1)^n}{6^n} \right)$

Additional Review Problems for These Sections

10. Determine if each statement below is always true or sometimes false.

(a) $\lim_{x\to 2} \frac{x-2}{x^2+x-6}$ is of an indeterminate form.

(b) $\lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^x = e$

(c) The integral $\int x^3\sqrt{1-x^2}\, dx$ can be evaluated by trigonometric substitution by setting $x = \sin x$.

(d) $\sin(\cos^{-1}(x)) = \tan(x)$.

(e) For the rational expression $\frac{x}{(x+10)(x-10)^2}$, the partial fraction decomposition is of the form $\frac{A}{x+10} + \frac{B}{(x-10)^2}$.

(f) For the rational expression $\frac{2x+3}{x^2(x+2)^2}$, the partial fraction decomposition is of the form $\frac{A}{x^2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$.

(g) If a sequence $\{a_n\}$ converges to a finite number L, then either the least upper bound or the greatest lower bound for $\{a_n\}$ is equal to L.

(h) If a sequence $\{a_n\}$ has both an upper bound and a lower bound, then $\{a_n\}$ converges.

(i) If a sequence $\{a_n\}$ does not have an upper bound, then $\{a_n\}$ diverges.

(j) The sum of two convergent geometric series is also convergent.

(k) The difference of two divergent series is also divergent.

(l) If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

(m) The integral $\int_{-1}^{1} \frac{1}{x^2}\, dx$ can be evaluated using the Fundamental Theorem of Calculus.

(n) The integral $\int_1^\infty \frac{1}{x^p}\, dx$ converges when $p \geq 1$.

11. Evaluate each integral using any method we have learned.

(a) $\int \frac{2x+1}{x^2-7x+12}\, dx$

(b) $\int \frac{8\, dx}{x^2\sqrt{4-x^2}}$

(c) $\int \frac{8\, dx}{(4x^2+1)^2}$

(d) $\int \frac{1}{(x+1)(x^2+1)}\, dx$ 12. Use L'Hopital's rule to evaluate the following limits.

(a) $\lim_{x\to\infty} (\ln x)^{\frac{1}{x^2+1}}$

(b) $\lim_{x\to 0^+} (\ln x)^x$

(c) $\lim_{x\to 0} \left[\frac{1}{x} - \cot x\right]$

(d) $\lim_{x\to\infty} \left[\cos\left(\frac{1}{x}\right)\right]^x$

13. Find values of a and b so that

$$\lim_{x\to 0} \frac{\cos(ax) - b}{2x^2} = -4.$$

14. Determine whether the sequences converge or diverge. Find the limit of each convergent sequence.

(a) $\left\{\left(1 - \frac{1}{n^2}\right)^n\right\}$

(b) $\left\{(10n)^{\frac{1}{n}}\right\}$

(c) $\left\{\frac{2n+(-1)^n}{4+3n}\right\}$

15. Find a formula for the nth term of the sequence. Then, determine whether the sequences converge or diverge. Find the limit of each convergent sequence.

(a) $\{1, -1, 1, -1, 1, -1, \ldots\}$

(b) $\{\sqrt{5}-\sqrt{4}, \sqrt{6}-\sqrt{5}, \sqrt{7}-\sqrt{6}, \sqrt{8}-\sqrt{7}, \sqrt{9}-\sqrt{8}, \sqrt{10}-\sqrt{9}, \ldots\}$

(c) $\left\{\sin\left(\frac{\sqrt{2}}{5}\right), \sin\left(\frac{\sqrt{3}}{10}\right), \sin\left(\frac{\sqrt{4}}{17}\right), \sin\left(\frac{\sqrt{5}}{26}\right), \sin\left(\frac{\sqrt{6}}{37}\right), \sin\left(\frac{\sqrt{7}}{50}\right), \ldots\right\}$

16. Determine if each integral below converges or diverges, and evaluate the convergent integrals.

(a) $\int_1^\infty \frac{\ln(x)}{x^2}\, dx$

(b) $\int_7^\infty \frac{dx}{x^2-x}$

(c) $\int_6^\infty \frac{3t^2}{\sqrt{t^3-8}}\,dt$

(d) $\int_2^6 \frac{3t^2}{\sqrt{t^3-8}}\,dt$

(e) $\int_2^\infty \frac{3t^2}{\sqrt{t^3-8}}\,dt$

17. Determine if each infinite series converges or diverges. If it converges, find the sum.

(a) $\sum_{n=0}^\infty e^{-3n}$

(b) $\sum_{n=1}^\infty \frac{4^n+5^n}{9^{n-1}}$

(c) $\sum_{n=0}^\infty \ln\left(\frac{n+5}{n+6}\right)$

(d) $\sum_{n=0}^\infty \cos(5\pi n)$

(e) $\sum_{n=1}^\infty \left(\frac{1}{\sqrt{n+3}} - \frac{1}{\sqrt{n+5}}\right)$

Answers

2. (c), (e), (h), (j), (m), (o), (s), (u), (w), (x), (y) are true

3. (a) $\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| - \frac{x}{\sqrt{x^2+4}} + C$ (b) $\frac{1}{2}(x^2+1)e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

(c) $-\frac{1}{3} \cdot \frac{(1-x^2)^{3/2}}{x^3} + C$ (d) $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$ (e) $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$

(f) $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$ (partial fractions)

(g) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$ (by parts)

(h) $4\ln|x| - 2\ln(x^2+1) + \tan^{-1}(x) + C$ (partial fractions)

(i) $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right) + \frac{x\sqrt{25-x^2}}{2} + C$ (trig sub)

(j) $-\frac{1}{x+1} + \frac{1}{(x+1)^2} + C$ (k) $x + \ln|x+1| + C$

(l) $-2\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$

4. (a) 0, (b) e^2, (c) $-\frac{1}{8}$

5. (a) $\frac{9}{2}$, (b) $\frac{1}{4}$, (c) diverges

6. (a) l.u.b.=g.l.b.=0 (b) l.u.b.=$\frac{1}{5}$ and g.l.b.=$-\frac{1}{25}$
(c) l.u.b.=2 and g.l.b.=1

7. (a) $\frac{2}{3}$ (b) $e^{-1/8}$ (c) diverges (d) $\frac{1}{e^5}$

8. $\frac{31}{99}$

9. (a) ≈ 0.1899 (b) diverges (c) $\frac{1}{2}$ (d) diverges (e) $3\frac{6}{7}$

10. (a), (c), (i), (j), (l) are true

11. (a) $-7\ln|x-3| + 9\ln|x-4| + C$, (b) $\frac{-2\sqrt{4-x^2}}{x} + C$

(c) $2\tan^{-1}(2x) + \frac{4x}{4x^2+1} + C$, (d) $\frac{1}{2}\ln|x+1| + \frac{1}{2}\arctan x - \frac{1}{4}\ln|x^2+1| + C$

12. (a) 1, (b) 1, (c) 0, (d) 1

13. $a = \pm 4$, $b = 1$

14. (a) 1, (b) 1, (c) $\frac{2}{3}$

15. (a) $\{(-1)^{n+1}\}$ and diverges, (b) $\left\{\sqrt{n+4} - \sqrt{n+3}\right\}$ and converges to 0

(c) $\left\{\sin\left(\frac{\sqrt{n+1}}{1+(n+1)^2}\right)\right\}$ and converges to 0.

16. (a) 1; (b) $\ln\left(\frac{7}{6}\right)$; (c) diverges; (d) $8\sqrt{13}$; (e) diverges

17. (a) $\frac{e^3}{e^3-1}$, (b) $\frac{369}{20}$, (c) diverges, (d) diverges, (e) $\frac{1}{2} + \frac{1}{\sqrt{5}}$

2 Review of Sections 10.3–10.9

Content Recap

1. Terminology review: complete the following statements.

(a) A geometric series has the general form _____________.
The series converges when _____________ and diverges when _____________.

(b) A p-series has the general form _____________. The series converges when _____________ and diverges when _____________. To show these results, we can use the _____________ test.

(c) The harmonic series has the form _____________, and it _____________.

(d) If you want to show a series converges, compare it to a _____________ series that also converges. If you want to show a series diverges, compare it to a _____________ series that also diverges.

(e) If the direct comparison test does not have the correct inequality, you can instead use the _____________ _____________ test. In this test, if the limit is a _____________ number (not equal to _____________), then both series converge or both series diverge.

(f) In the ratio and root tests, the series will _____________ if the limit is less than 1 and _____________ if the limit is greater than 1. If the limit equals 1, then the test is _____________.

(g) If $\sum_k a_k$ is an alternating series, then it converges _____________ if $\sum_k |a_k|$ converges. It converges

_____________ if $\sum_k |a_k|$ diverges and (i) the limit of the terms is _____________ and (ii) the sequence of terms is _____________.

(h) If an alternating series converges, we can estimate the sum by adding the first n terms. Stopping after n terms will give us an error at most equal to the magnitute of the _____________ term in the sequence.

(i) If $\lim_{n\to\infty} a_n = 0$, then what, if anything, do we know about the series $\sum_n a_n$? _____________

(j) A power series has the general form: _____________. To find the radius of convergence R, use either the _____________ or _____________ test. The series converges _____________ when $|x - c| < R$. To find the interval of convergence, don't forget to check the _____________.

(k) A Taylor polynomial has the general form: _____________. The Taylor polynomial is the n^{th} _____________ _____________ of the Taylor series with general form: _____________.

(l) The Taylor remainder theorem says that $|R_n| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$, where M represents the maximum value of the _____________ derivative of f on the interval between x and a. The remainder term decreases when n _____________ or when x is _____________ to a.

(m) A MacLaurin Series is a Taylor series centered at _____________.

(n) Complete the formulas for the common MacLaurin series.

$$e^x = \sum_{k=0}^{\infty}$$

$$\ln(1+x) = \sum_{k=0}^{\infty}$$

$$\sin(x) = \sum_{k=0}^{\infty}$$

$$\cos(x) = \sum_{k=0}^{\infty}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty}$$

(o) Fill in the formulas for the derivatives and anti-derivatives of a power series.

$$\frac{d}{dx}\left[\sum_{k=0}^{\infty} a_k x^k\right] =$$

$$\int_0^x \left[\sum_{k=0}^{\infty} a_k t^k\right]\, dt =$$

Problems Similar to the Studio Worksheets

2. Determine if each of the following statements is always true or sometimes false.

(a) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3+1}}$ is a p-series with $p = \frac{3}{2}$.

(b) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$ converges when $p > 1$.

(c) To show a series $\sum_k a_k$ converges by the Basic Comparison Test, we should find a smaller series $\sum_k b_k$ that also converges.

(d) A limit of 0 or ∞ from the Limit Comparison Test may not give us a conclusive answer as to whether our series converges or diverges.

(e) To determine whether $\sum_{k=3}^{\infty} \frac{k}{k^3-10}$ converges or diverges, use the Basic Comparison Test with $\sum_{k=3}^{\infty} \frac{1}{k^2}$.

(f) If $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} < 1$, then the series $\sum_k a_k$ converges.

(g) We should use the root test if all of the terms are raised to the k^{th} power.

(h) We can use the root test to show that the p-series $\sum_k \frac{1}{\sqrt{k}}$ diverges.

(i) The ratio test would be inconclusive for the series $\sum_k \frac{k}{k^3+1}$.

(j) $(2k)! = 2k!$

(k) If an alternating series converges absolutely, then it also converges conditionally.

(l) If $\sum_k |a_k|$ converges, then the alternating series $\sum_k a_k$ also converges.

(m) If $\sum_k a_k$ is an alternating series and $\{|a_k|\}$ is a decreasing sequence, then $\sum_k a_k$ converges.

(n) If $\sum_k a_k$ is an alternating series and $\sum_k |a_k|$ diverges, then $\sum_k a_k$ cannot converge absolutely.

(o) If $\sum_k a_k$ is an alternating series and $\lim_{k\to\infty} |a_k| \neq 0$, then $\sum_k a_k$ diverges.

3. Determine whether the following series converge or diverge. Justify your answers using any of the tests we have discussed in class. Make sure that you (1) name the test and state the conditions needed for the test you are using, (2) show work for the test that requires some math, and (3) state a conclusion that explains why the test shows convergence or divergence.

(a) $\sum_{k=1}^{\infty} \frac{e^k}{4+e^{2k}}$

(b) $\sum_{k=1}^{\infty} \left(1 - \frac{3}{k}\right)^k$

(c) $\sum_{k=1}^{\infty} k \tan\left(\frac{1}{k}\right)$

(d) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$

(e) $\sum_{k=1}^{\infty} \frac{3^{2k}}{8^k-3}$

(f) $\sum_{k=1}^{\infty} \frac{k+2}{\sqrt{k^5+4}}$

(g) $\sum_{k=1}^{\infty} \frac{k+3}{\sqrt{k^2+1}}$

(h) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots$

(i) $\sum_{k=1}^{\infty} \frac{\ln k}{k^4}$

(j) $\sum_{k=1}^{\infty} \frac{(2k)^k}{k!}$

(k) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{2k^2}$

(l) $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{4^n 2^n n!}$

4. Suppose $r > 0$. Find the values of r, if any, for which $\sum_{k=1}^{\infty} \frac{r^k}{k^r}$ converges.

5. Determine whether the following alternating series converge absolutely, converge conditionally, or diverge. Justify your answers using the tests we discussed in class.

(a) $\sum_{k=2}^{\infty} (-1)^{k+1} \frac{3k}{\sqrt{k^3+4}}$

(b) $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k^4-1}$

(c) $\sum_{k=0}^{\infty} (-1)^k \frac{k}{5^k+2^k}$

(d) $\sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k \sqrt{\ln \ln k}}$

6. Find the radius and interval of convergence of the following power series:

(a) $\sum_{k=2}^{\infty} \left(\frac{k}{k-1}\right) \frac{(x+2)^k}{2^k}$ (b) $\sum_{n=1}^{\infty} \frac{(3x+2)^n}{\sqrt{n}}$

7. Find a power series representation for the function $f(x) = \frac{x}{4+x^4}$. For what values of x does the series converge?

8. Find the third degree Taylor polynomial of the function $f(x) = \tan^{-1}(x)$ in powers of $x - 1$.

9. Use a Taylor polynomial to estimate the value of $\sqrt{e}$ with an error of at most 0.01. HINT: Choose $a = 0$ and use the fact that $e < 3$.

10. For what values of x can we replace $\cos x$ with $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ within an error range of no more that 0.001?

11. Use the MacLaurin series for $f(x) = \frac{1}{1-x}$ to find a power series representation of the function

$$g(x) = \frac{x}{(1-x)^3}.$$

HINT: You will need to differentiate.

12. Find $f^{(7)}(0)$ for the function $f(x) = x\sin(x^2)$.

13. Find a power series (i.e., MacLaurin series) representation for the following functions. When is your series valid?
(a) $f(x) = \frac{3x}{2+4x}$

(b) $g(x) = xe^{-x}$

14. Find a MacLaurin series for the function $f(x) = \tan^{-1} x$.

15. Find the sum of the series:

$$\frac{\pi}{2} - \frac{\pi^3}{8 \cdot 3!} + \frac{\pi^5}{32 \cdot 5!} + ... + (-1)^n \frac{\pi^{2n+1}}{2^{2n+1}(2n+1)!} + ...$$

16. Use a MacLaurin series to estimate $\int_0^1 e^{-x^2} dx$ within an error of no more than 0.01.

Additional Review Problems on These Sections

17. Let $\{a_n\}$ and $\{b_n\}$ be a sequences of non-negative terms. Are the following statements *always* true or sometimes false?

(a) If $\lim_{n\to\infty} a_n = L$, then the series $\sum_n a_n = L$.

(b) If $\lim_{n\to\infty} a_n = 0$, then $\{a_n\}$ converges to 0.

(c) If $\lim_{n\to\infty} a_n = 0$, then $\sum_n a_n$ converges.

(d) If $\sum_n a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

(e) If $\sum_n a_n$ diverges, then $\lim_{n\to\infty} a_n \neq 0$.

(f) If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_n a_n$ diverges.

(g) If $\lim_{n\to\infty} a_n \neq 0$, then $\{a_n\}$ diverges.

(h) If $\int_1^\infty f(x)dx = L$, where $0 < L < \infty$, then $\sum_n f(n) = L$.

(i) If $\sum_n b_n$ converges and $a_n > b_n$ for all $n \geq 1$, then $\sum_n a_n$ also converges.

(j) If $\sum_n b_n$ diverges and $a_n > b_n$ for all $n \geq 1$, then $\sum_n a_n$ also diverges.

(k) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_n b_n$ diverges, then $\sum_n a_n$ also diverges.

18. Determine if each statement below is always true or sometimes false.

(a) If $p > 1$ the alternating p-series $\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n^p}$ converges conditionally.

(b) The alternating harmonic series $\sum_{n=1}^\infty (-1)^{n+1}\frac{1}{n}$ is conditionally convergent.

(c) If $\sum_{n=1}^{\infty} a_n$ is a convergent series of nonnegative numbers, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ also converges.

(d) If $\sum_{n=1}^{\infty} a_n$ is a convergent series of nonnegative numbers, then $\sum_{n=1}^{\infty} \sin(a_n)$ also converges.

(e) The series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges for any $p > 1$.

(f) The series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ diverges for any $p \leq 1$.

(g) Let $\{a_n\}$ be a positive sequence and $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ be a finite number. If a series $\sum_{n=1}^{\infty} a_n$ converges, then $L < 1$.

(h) Let $\{a_n\}$ be a positive sequence and $L = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$ be a finite number. If a series $\sum_{n=1}^{\infty} a_n$ diverges, then $L \geq 1$.

(i) If the radius of convergence of a power series is 0, then the power series diverges everywhere.

(j) If a power series $\sum_{n=0}^{\infty} a_n x^n$ converges in $(-1, 1)$, then its radius of convergence is 1.

(k) Every Taylor series is a power series.

(l) The fifth degree Taylor polynomial for $\cos(x)$ about $x = 0$ is $1 - \frac{x^2}{2} + \frac{x^4}{4!}$.

(m) For any Taylor polynomial, the error in the approximation is no more than the magnitude of the $(n+1)^{st}$ term.

19. Let $f(x) = \int_0^x t\sin(t^3)dt$. Use a MacLaurin series to find $f^{(11)}(0)$.

20. (a) Estimate $\cos\left(\frac{\pi}{12}\right)$ using a fourth-degree Taylor polynomial.

(b) Find a MacLaurin series for the function $g(x) = \int_0^x \frac{\sin(t/2)}{2t}dt$.

21. Determine if the alternating series converges absolutely or converges conditionally. Justify your answer fully by: (1) name the test and state the conditions needed for the test you are using, (2) show work for the test that requires some math, and (3) state a conclusion that explains why the test shows convergence or divergence.

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$

(b) $\sum_{n=1}^{\infty}(-5)^{-n}$

22. Determine whether the given series converges or diverges. Make sure that you (1) name the test and state the conditions needed for the test you are using, (2) show work for the test that requires some math, and (3) state a conclusion that explains why the test shows convergence or divergence.

(a) $\sum_{n=2}^{\infty} \frac{n+1}{\sqrt{n^3-2}}$

(b) $\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$ (Hint: Limit Comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.)

(c) $\sum_{n=3}^{\infty} \frac{10}{n \ln n \ln(\ln n)}$

(d) $\sum_{n=1}^{\infty} ne^{-n}$

(e) $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$

(f) $\sum_{n=1}^{\infty} \left(\frac{\sin n}{1+n}\right)^n$

(g) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{2\cdot 3^{n-1}}$

(h) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$

23. Find the radius and interval of convergence of each power series below.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n(2x-4)^n}{n^2}$

(b) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n x^n$

Answers

2. (b), (d), (g), (i), (l), (n), and (o) are true

3. (a), (d), (f), (h), (i), (k), and (l) converge

4. converges when $0 < r < 1$

5. (a) and (d) converge conditionally, (b) and (c) converge absolutely

6. (a) $R = 2$, $I.C. = (-4, 0)$, (b) $R = \frac{1}{3}$, $I.C. = \left[-1, -\frac{1}{3}\right)$

7. $\sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+1}}{4^{k+1}}$, $|x| < 4^{1/4}$

8. $\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$

9. 1.6458

10. $x \in (-0.9467, 0.9467)$

11. $\frac{1}{2}\sum_{k=2}^{\infty} k(k-1)x^{k-1}$

12. -840

13. (a) $3\sum_{k=0}^{\infty}(-1)^k 2^{k-1}x^{k+1}$, $|x| < \frac{1}{2}$ (b) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k!}$, $x \in \Re$

14. $\sum_{k=0}^{\infty}(-1)^k \cdot \frac{x^{2k+1}}{2k+1}$, $|x| < 1$

15. 1

16. ≈ 0.743

17. Statements (b), (d), (f), and (j) are true.

18. (b), (c), (d), (e), (f), (h), (k), and (l) are true

19. $-\frac{10!}{6}$

20. (a) 0.966, (b) $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2^{2k+2}(2k+1)!(2k+1)}$

21. (a) converges conditionally; (b) converges absolutely

22. (b), (d), (e), (f), (g), and (h) converge

23. (a) $R = \frac{1}{2}$, $I.C. = \left[\frac{3}{2}, \frac{5}{2}\right]$; (b) $R = 1$, $I.C. = (-1, 1)$

3 Review of Volumes of Revolution: Sections 6.1–6.2

Additional Practice Problems on Volumes

1. Find the volume of the solid generated by revolving the region bounded by the curve $y = \sin(x)$, the x-axis, and the lines $x = 0$, $x = \pi/2$ about the y-axis.

2. Find the volume of the solid generated when the region bounded by the curves $y = 4 - x^2$ and $y = 2 - x$ is revolved about the x-axis.

3. Find the volume of the solid generated when the region bounded by the curves $y = x^2 - 4$ and $y = 2x - x^2$ is revolved about the line (a) $y = -4$ and (b) $x = 2$.

4. Use the method of cylindrical shells to find the volume of the solid generated when the region bounded by the curve $y = \sqrt{x}$, the x-axis, and the line $x = 9$ is revolved about the x-axis.

Answers to Additional Problems on Volumes

1. 2π

2. $\frac{108\pi}{5}$ cubic units

3. (a) 45π cubic units, (b) 27π cubic units

4. $\frac{81\pi}{2}$ cubic units

4 Problems From a Previous Final Exam Study Packet

The Problem Set

1. Sum the series
$$\sum_{k=2}^{\infty} \frac{4^{2k}-1}{17^{k-1}}.$$

2. Find the sum of the series
$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+3)}.$$

3. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

   (a) $\sum_{k=1}^{\infty} \frac{e^k}{(1+4e^k)^{3.2}}$

   (b) $\sum_{k=2}^{\infty} \left(\frac{k-5}{k}\right)^{k^2}$

   (c) $\sum_{k=1}^{\infty} \frac{k^2 \cdot 2^{k+1}}{k!}$

   (d) $\sum_{k=1}^{\infty} \frac{1}{1+2+3+\ldots+k}$

4. Find the third degree Taylor polynomial of the function $f(x) = \tan^{-1}(x)$ in powers of $x-1$.

5. Use a Taylor polynomial to estimate the value of $\sqrt{e}$ with an error of at most 0.01. HINT: Choose $a = 0$ and use the fact that $e < 3$.

6. Use the MacLaurin series for $f(x) = \frac{1}{1-x}$ to find a power series representation of the function
$$g(x) = \frac{x}{(1-x)^3}.$$
HINT: You will need to differentiate.

7. Find the radius and interval of convergence of the power series
$$\sum_{k=1}^{\infty} \frac{5^k}{\sqrt{k}}(3-2x)^k.$$

8. Determine whether each of the alternating series below converge absolutely, converge conditionally, or diverge. Use the convergence tests from class to justify your answer.

   (a)
$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\ln k}{k^4}$$

   (b)
$$\sum_{k=2}^{\infty}(-1)^{k}\frac{4k^2}{k^3+1}$$

9. Does the alternating series $\sum_{k=1}^{\infty}\frac{(-1)^k}{k+\sqrt{k}}$ converge absolutely, converge conditionally, or diverge?

10. Evaluate each integral below using any of the methods we have learned.

    (a) $\int \frac{\sin^3 x}{\cos x}dx$

    (b) $\int \frac{x}{\sqrt{x^2+2x-3}}dx$

    (c) $\int \frac{\cos x}{4+\sin^2 x}dx$

    (d) $\int \frac{1}{x(x^2+x+1)}dx$

11. Evaluate the improper integral if it converges, or show that the integral diverges.
$$\int_0^3 \frac{x}{(x^2-1)^{2/3}}dx$$

12. Does the integral
$$\int_0^{\infty}\frac{dx}{e^x+e^{-x}}$$
converge or diverge? (HINT: Use the Integral Comparison Test.)

13. Find the area bounded between the curves $y = 2\cos x$ and $y = \sin(2x)$ on the interval $[-\pi, \pi]$.

14. Evaluate the integrals:

    (a) $\int \left(\sqrt{x} - \frac{1}{x^2}\right)^2 dx$

    (b) $\int \frac{\log_3 x^4}{x} dx$

    (c) $\int \frac{\sec(e^{-4x})}{e^{4x}} dx$

    (d) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$

    (e) $\int \frac{x^2}{(ax^3+b)^2} dx$

    (f) $\int_{-5}^0 \left(x\sqrt{4-x}\right) dx$

15. Find the general solution to the equation:

$$(y \ln x)y' = \frac{y^2+1}{x}.$$

16. Evaluate the following integrals.

    (a) $\int x^5 \ln(x) dx$

    (b) $\int x^3 e^{x^2} dx$

    (c) $\int (\ln x)^2 dx$

17. Evaluate the following integrals.

    (a) $\int x \tan^{-1}(x) dx$

    (b) $\int \frac{\cos^3(x)}{\sin x} dx$

    (c) $\int \sqrt{9-x^2} dx$

18. Find the area of the region bounded by the curves $y = 5x + 1$ and $y = x^2 + 3x - 2$.

19. Find the volume of the solid generated by revolving the region bounded by the curve $y = \sin(x)$, the x-axis, and the lines $x = 0$, $x = \pi/2$ about the y-axis.

20. Let $f(x) = \int_0^x t \sin(t^3) dt$. Use a MacLaurin series to find $f^{(11)}(0)$.

21. Evaluate $\int \left(e^{-5x} + \frac{1}{1+4x^2}\right) dx$.

22. Find the sum of the series:

$$1 - \frac{\pi^2}{9 \cdot 2!} + \frac{\pi^4}{81 \cdot 4!} + ... + (-1)^n \frac{\pi^{2n}}{3^{2n}(2n)!} + ...$$

23. Evaluate the integrals:

   (a) $\int \frac{x^2}{\sqrt{4-x^6}} dx$

   (b) $\int_0^2 x|2x-1| dx$

24. Find $F'(2)$ for the function

$$F(x) = \int_{\frac{8}{x}}^{x^2} \left(\frac{t}{1-\sqrt{t}} \right) dt.$$

25. Find the area bounded by the curves $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$.

26. Evaluate the integrals:

   (a) $\int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$

   (b) $\int \frac{1}{\ln(x^x)} dx$

27. Evaluate the following integrals:

   (a) $\int \frac{e^{2x}}{\sqrt{4-3e^{2x}}} dx$

   (b) $\int_{-3}^{-2} \frac{dx}{\sqrt{4-(x+3)^2}}$

28. Solve the initial value problem:

$$y' = x\sqrt{\frac{1-y^2}{1-x^2}}, \quad y(0) = 0.$$

29. Evaluate the following integrals.

   (a) $\int \sin^5(2x)\cos^3(2x) dx$

   (b) $\int \tan^4(x) dx$

   (c) $\int e^{2x} \sin(3x) dx$

   (d) $\int \frac{x^2}{(x^2+4)^{3/2}} dx$

   (e) $\int \frac{\sqrt{1-x^2}}{x^4} dx$

   (f) $\int \frac{x}{(4-x^2)^{3/2}} dx$

   (g) $\int \frac{dx}{e^x\sqrt{e^{2x}-9}}$

30. Evaluate the following integrals.

   (a) $\int \frac{x+3}{(x-1)(x^2-4x+4)} dx$

(b) $\int \frac{x+4}{x^3+x} dx$

31. Evaluate each of the following integrals.

   (a) $\int \frac{x+2}{x+1} dx$

   (b) $\int \sqrt{25 - x^2} dx$

   (c) $\int \tan^3(x) \sec^4(x) dx$

   (d) $\int x \tan^{-1}(x) dx$

   (e) $\int \frac{dx}{x\sqrt{1+x^2}}$

   (f) $\int \frac{x+1}{x^2(x-1)} dx$

   (g) $\int \frac{x+1}{x^2-4x+8} dx$

32. Evaluate the improper integrals if they converge, or show that the integral diverges.

   (a) $\int_1^3 \frac{1}{(x^2-1)^{3/2}} dx$

   (b) $\int_0^\infty x^2 e^{-2x} dx$

33. For what values of p does the integral $\int_4^\infty \frac{dx}{x(\ln x)^p}$ converge?

34. Find the area bounded by the curve $y = \frac{1}{x^2+9}$, the x-axis, and $x \geq 0$.

35. Use series to write the repeating decimal 0.31313131... as a rational number.

36. Find the sum of each convergent series below, or explain why the series diverges.

$$\sum_{k=7}^{\infty} \frac{1}{(k-3)(k+1)}, \quad \sum_{k=0}^{\infty} (-1)^k, \quad \sum_{k=2}^{\infty} \frac{2^k+1}{3^{k+1}}$$

37. Determine if each series below converges or diverges. JUSTIFY YOUR ANSWER FULLY using either the nth term divergence test or the integral test.

   (a)

$$\sum_{k=1}^{\infty} \frac{e^k}{4+e^{2k}}$$

   (b)

$$\sum_{k=1}^{\infty} \frac{5k^2+8}{7k^2+6k+1}$$

38. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

(a) $\sum_{k=1}^{\infty} \frac{3^{2k}}{8^k-3}$

(b) $\sum_{k=1}^{\infty} \frac{k+2}{\sqrt{k^5+4}}$

(c) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$

39. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

    (a) $\sum_{k=1}^{\infty} \frac{(2k)^k}{k!}$

    (b) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{2k^2}$

    (c) $\sum_{k=1}^{\infty} k \tan\left(\frac{1}{k}\right)$

    (d) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots$

40. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

    (a) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$

    (b) $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{4^n 2^n n!}$

41. Suppose $r > 0$. Find the values of r, if any, for which $\sum_{k=1}^{\infty} \frac{r^k}{k^r}$ converges.

42. Determine whether the following alternating series converge absolutely, converge conditionally, or diverge. Justify your answers using the tests we discussed in class.

    (a) $\sum_{k=2}^{\infty}(-1)^{k+1}\frac{3k}{\sqrt{k^3+4}}$

    (b) $\sum_{k=2}^{\infty}(-1)^k\frac{k}{k^4-1}$

    (c) $\sum_{k=0}^{\infty}(-1)^k\frac{k}{5^k+2^k}$

    (d) $\sum_{k=2}^{\infty}(-1)^k\frac{1}{k\ln k\sqrt{\ln\ln k}}$

43. Find the radius and interval of convergence of the power series $f(x) = \sum_{k=1}^{\infty} \frac{2^k}{k+1}(x-3)^k$.

44. For what values of x can we replace $\cos x$ with $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ within an error range of no more that 0.001?

45. Find $f^{(7)}(0)$ for the function $f(x) = x\sin(x^2)$.

46. Use a MacLaurin series to estimate $\int_0^1 e^{-x^2}dx$ within an error of no more than 0.01.

47. Find the volume of the solid generated when the region bounded by the curves $y = 4 - x^2$ and $y = 2 - x$ is revolved about the x-axis.

48. Find the volume of the solid generated when the region bounded by the curves $y = x^2 - 4$ and $y = 2x - x^2$ is revolved about the line $y = -4$.

49. Find the volume of the solid generated when the region bounded by the curves $y = x^2 - 4$ and $y = 2x - x^2$ is revolved about the line $x = 2$.

50. Use the method of cylindrical shells to find the volume of the solid generated when the region bounded by the curve $y = \sqrt{x}$, the x-axis, and the line $x = 9$ is revolved about the x-axis.

51. Find a power series representation for the following functions. When is your series valid?

    (a) $f(x) = \frac{3x}{2+4x}$

    (b) $g(x) = \int_0^x \frac{\sin(t/2)}{2t} dt$

    (c) $h(x) = \tan^{-1}(x)$

52. (a) Estimate $\cos 15°$ using a fourth-degree Taylor polynomial.

    (b) Estimate $\int_0^1 e^{-2x^2} dx$ within an error of 0.01.

Answers

1. $255\frac{15}{16}$

2. $\frac{1}{3}$

3.
(a) Converges by the integral test
(b) Converges by the root test
(c) Converges by the ratio test
(d) Converges by the basic comparison test, or using telescoping series

4. $P_3(x) = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$

5. $\sqrt{e} \approx f(0.5) = 1 + 0.5 + \frac{(0.5)^2}{2} + \frac{(0.5)^3}{6} = 1.6458.$

6. $\frac{1}{2}\sum_{k=2}^{\infty} k(k-1)x^{k-1}$, $|x| < 1$

7. $R = \frac{1}{10}$, IC$=\left(\frac{7}{5}, \frac{8}{5}\right]$

8.
(a) Converges absolutely by the basic comparison test
(b) Converges conditionally by the limit comparison and alternating series tests

9. converges conditionally

10.
(a) $-\ln|\cos x| + \frac{1}{2}\cos^2 x + C$
(b) $\sqrt{x^2+2x-3} - \ln|\frac{x+1+\sqrt{x^2+2x-3}}{2}| + C$
(c) $\frac{1}{2}\tan^{-1}\left(\frac{\sin x}{2}\right) + C$
(d) $\ln|x| - \frac{1}{2}\ln(x^2+x+1) - \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

11. Converges to $\frac{9}{2}$

12. Converges (compare to $\int_0^\infty \frac{1}{e^x} dx$)

13. 8

14.

(a) $\frac{1}{2}x^2 + \frac{4}{\sqrt{x}} - \frac{1}{3x^3} + C$

(b) $\frac{2}{\ln 3}(\ln x)^2 + C$

(c) $-\frac{1}{4}\ln|\sec(e^{-4x}) + \tan(e^{-4x})| + C$

(d) $\frac{2}{3}$

(e) $-\frac{1}{3a(ax^3+b)} + C$

(f)$-\frac{506}{15}$

15. $y^2 = k(\ln x)^2 - 1$, where $k = e^{2C}$

16.
(a) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$

(b) $\frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2} + C$

(c) $x(\ln x)^2 - 2x\ln x + 2x + C$

17.
(a) $\frac{x^2 \tan^{-1} x}{2} - \frac{x}{2} + \frac{\tan^{-1} x}{2} + C$

(b) $\ln|\sin x| - \frac{1}{2}\sin^2 x + C$

(c) $\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x\sqrt{9-x^2}}{2} + C$

18. $\frac{32}{3}$

19. 2π

20. $-\frac{10!}{6}$

21. $-\frac{1}{5}e^{-5x} + \frac{1}{2}\tan^{-1}(2x) + C$

22. $\frac{1}{2}$

23.
(a) $\frac{1}{3}\sin^{-1}\left(\frac{x^3}{2}\right) + C$

(b) $\frac{41}{12}$

24. -24

25. $\frac{37}{12}$

26. $-\sec\left(\frac{1}{x}\right) + C$, $\ln|\ln x| + C$

27. $-\frac{1}{3}\sqrt{4-3e^{2x}}$, $\frac{\pi}{6}$

28. $y = \sin(-\sqrt{1-x^2}+1)$

29.
(a) $\frac{1}{12}\sin^6(2x) - \frac{1}{16}\sin^8(2x) + C$

(b) $\frac{1}{3}\tan^3(x) - \tan(x) + x + C$

(c) $\frac{2}{13}e^{2x}\sin(3x) - \frac{3}{13}e^{2x}\cos(3x) + C$

(d) $\ln|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}| - \frac{x}{\sqrt{x^2+4}} + C$

(e) $-\frac{1}{3}\cdot\frac{(1-x^2)^{3/2}}{x^3} + C$

(f) $\frac{1}{\sqrt{4-x^2}} + C$

(g) $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$

30.
(a) $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$

(b) $4\ln|x| - 2\ln(x^2+1) + \tan^{-1}(x) + C$

31.
(a) $x + \ln|x+1| + C$

(b) $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)+\frac{x\sqrt{25-x^2}}{2}+C$

(c) $\frac{1}{4}\tan^4(x)+\frac{1}{6}\tan^6(x)+C$

(d) $\frac{x^2}{2}\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+C$

(e) $-\ln\left|\frac{\sqrt{1+x^2}}{x}+\frac{1}{x}\right|+C$

(f) $-2\ln|x|+\frac{1}{x}+2\ln|x-1|+C$

(g) $\frac{1}{2}\ln|x^2-4x+8|+\frac{3}{2}\tan^{-1}\left(\frac{x-2}{2}\right)+C$

32. (a) diverges, (b) converges to $\frac{1}{4}$

33. converges when $p>1$

34. $\frac{\pi}{6}$ units^2

35. $\frac{31}{99}$

36. (a) ≈ 0.1899, (b) diverges (look at the sequence of partial sums), (c) $\frac{1}{2}$

37. (a) converges by the integral test, (b) diverges by the nth term test

38. (a) diverges by basic comparison, (b) converges by basic comparison, (c) converges by the integral test

39. (a) diverges by the ratio test, (b) converges by the root test, (c) diverges by the nth term test, (d) converges by limit comparison (or, you can show it is telescoping)

40. (a) converges by limit comparison, (b) converges by ratio test

41. converges when $0<r<1$

42. (a) converges conditionally, (b) converges absolutely, (c) converges absolutely, (d) converges conditionally

43. $\left[\frac{5}{2},\frac{7}{2}\right)$

44. $x\in(-0.9467, 0.9467)$

45. -840

46. ≈ 0.743

47. $\frac{108\pi}{5}$ cubic units

48. 45π cubic units

49. 27π cubic units

50. $\frac{81\pi}{2}$ cubic units

51.
(a) $3\sum_{k=0}^{\infty}(-1)^k 2^{k-1}x^{k+1}$, valid for $|x| < \frac{1}{2}$
(b) $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2^{2k+2}(2k+1)!(2k+1)}$, valid for $x \neq 0$
(c) $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1} + C$, valid for $|x| < 1$

52. (a) 0.966; (b) approximately 0.6165 (stop at $k = 5$)

5 A Few Extra Practice Problems that May Appear on the Final Exam – KNOW THESE

1. Evaluate the following improper integral:

$$I = \int_{e^2}^{\infty} \frac{\ln(\ln(x))}{x(\ln x)^{m+1}} dx, m > 0.$$

2. Evaluate the following indefinite integral:

$$I = \int \frac{dx}{1 + e^x}.$$

3. Evaluate the next limit

$$L = \lim_{\alpha \to 0^+} \left(\frac{1 - e^{-\alpha v}}{\alpha}\right)^x, \alpha > 0, v > 0.$$

HINT: Consider the Taylor series expansion of the exponential function.